

Name:
ID:
Section:

Formula Sheet-Quiz 2:

Permittivity of free space: $\epsilon_0 = 8.854 \times 10^{-12} \frac{F}{m}$

Permeability of free space: $\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}$

Maxwell's Equations for Electrostatics:

Integral form

Differential Form

$$\oint D \cdot dS = Q_{enc} \text{ (Gauss's Law)}$$

$$\nabla \cdot D = \rho_v \Rightarrow \oint D \cdot dS = \int \nabla \cdot D dv \text{ (Divergence Theorem)}$$

$$\oint E \cdot dL = 0$$

$$\nabla \times E = 0$$

Uniform Charge Distributions

Uniform line charge ($\rho_l = \text{constant}$)

$$Q = \int_L \rho_l dl' = \rho_l \int_L dl' = \rho_l L_o \Rightarrow \rho_l = \frac{Q}{L_o}$$

(L_o = total line length)

Uniform surface charge ($\rho_s = \text{constant}$)

$$Q = \int_S \rho_s ds' = \rho_s \int_S ds' = \rho_s A_o \Rightarrow \rho_s = \frac{Q}{A_o}$$

(A_o = total surface area)

$$Q = \int_V \rho_v dV$$

Uniform volume charge ($\rho_v = \text{constant}$)

$$Q = \int_V \rho_v dv' = \rho_v \int_V dv' = \rho_v V_o \Rightarrow \rho_v = \frac{Q}{V_o}$$

(V_o = total volume)

The electric field intensity resulting from a line charge (ρ_L) / surface charge (ρ_s) / volume charge (ρ_v) distribution is:

$$\mathbf{E} = \int \frac{\rho_L dl}{4\pi\epsilon_0 R^2} \mathbf{a}_R \quad (\text{Line Charge})$$

$$\mathbf{E} = \int \frac{\rho_s dS}{4\pi\epsilon_0 R^2} \mathbf{a}_R \quad (\text{Surface Charge})$$

$$\mathbf{E} = \int \frac{\rho_v dV}{4\pi\epsilon_0 R^2} \mathbf{a}_R \quad (\text{Volume Charge})$$

Where $\mathbf{R} = R\mathbf{a}_R$ is the vector from the charge distribution to the location where the field is desired

The electric field intensity from:

a- A point charge Q located at the origin and multiple charges located at points with positions $R_1, R_2 \dots R_N$

$$\text{for a single Charge: } E = \frac{Q}{4\pi\epsilon_0 r^2} a_r \text{ V/m} \quad \text{for N Charges: } E = \frac{1}{4\pi\epsilon} \sum_{i=1}^N \frac{q_i (R - R_i)}{|R - R_i|^3} \text{ V/m}$$

b- An infinite line of charge ρ_L is:

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \mathbf{a}_\rho$$

c- A ring line of charges of radius "a" lying on the x-y plane and centered at (0, 0, 0) at P(0, 0, h) is:

$$\mathbf{E} = \frac{\rho_L a h}{2\epsilon_0 (a^2 + h^2)^{3/2}} \mathbf{a}_z$$

d- An infinite sheet charge is: $\mathbf{E} = \frac{\rho_s}{2\epsilon_0} \mathbf{a}_N$

The electric force on a charge q due to an electric field intensity is:

$$\mathbf{F}_e = q\mathbf{E}$$

-The force that charge Q_1 acts on Q_2 is

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \mathbf{a}_{12}$$

Electric flux density D is:

$$\mathbf{D} = \epsilon\mathbf{E} = \epsilon_r \epsilon_0 \mathbf{E}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \epsilon_0 \chi_e \mathbf{E}$$

$$= \epsilon_0 (1 + \chi_e) \mathbf{E} = \epsilon \mathbf{E}$$

The total electric flux passing through a surface S is:

$$\psi = \int \mathbf{D} \cdot d\mathbf{S}$$

The electric potential due to a charge distribution is:

$$V = \sum_{i=1}^N \frac{Q_i}{4\pi\epsilon_0 r_i} = \int \frac{dQ}{4\pi\epsilon_0 r}$$

The electric potential between a pair of points a and b in an electric field is:

$$V_{ba} = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$$

The electric field is related to V:

$$\mathbf{E} = -\nabla V$$

The current through an area is:

$$I = \int \mathbf{J} \cdot d\mathbf{S} = \int \sigma \mathbf{E} \cdot d\mathbf{S}$$

Joule's Law indicates the amount of dissipated power: $P = \int \mathbf{E} \cdot \mathbf{J} dv$

Poisson's Equation: $\nabla^2 V = -\frac{\rho_v}{\epsilon}$

In a charge-free medium: $\nabla^2 V = 0$

The electric field at point P with Cartesian coordinates (0, 0, h) due to a circular disk of radius a and uniform charge density ρ_s residing in the x-y plane

$$\mathbf{E} = \frac{\rho_s}{2\epsilon_0} \left[1 - \frac{h}{\sqrt{a^2 + h^2}} \right] \mathbf{a}_z$$

Divergence:

Cartesian

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cylindrical

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Spherical

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

Gradient:

Cartesian

$$\nabla V = \frac{\partial V}{\partial x} \mathbf{a}_x + \frac{\partial V}{\partial y} \mathbf{a}_y + \frac{\partial V}{\partial z} \mathbf{a}_z$$

Cylindrical

$$\nabla V = \frac{\partial V}{\partial \rho} \mathbf{a}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \mathbf{a}_\phi + \frac{\partial V}{\partial z} \mathbf{a}_z$$

Spherical

$$\nabla V = \frac{\partial V}{\partial r} \mathbf{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \mathbf{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \mathbf{a}_\phi$$

Laplacian:

Cartesian

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

Curl:

Cartesian

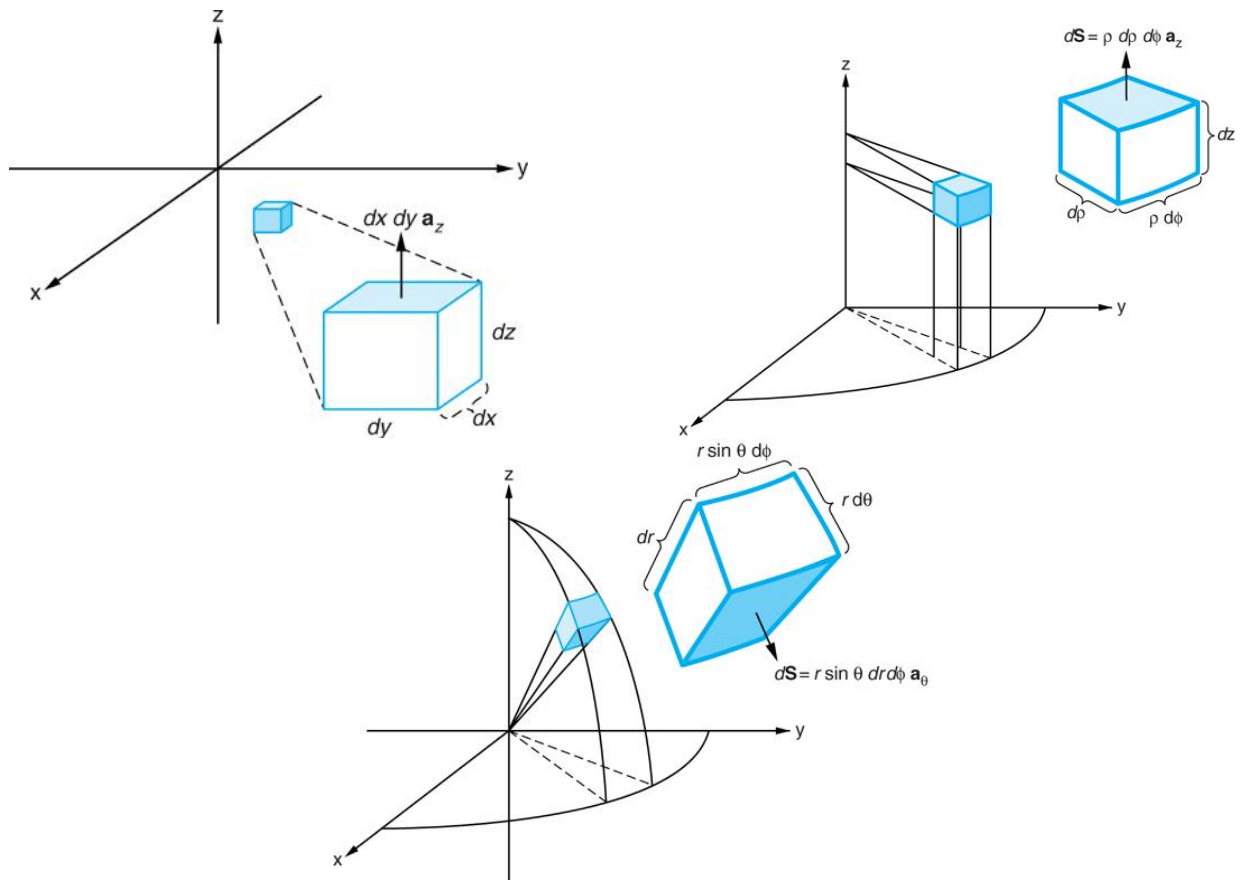
$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \mathbf{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \mathbf{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \mathbf{a}_z$$

Cylindrical

$$\nabla \times \mathbf{A} = \left[\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right] \mathbf{a}_\phi + \frac{1}{\rho} \left[\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \right] \mathbf{a}_z$$

Spherical

$$\nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \left[\frac{\partial(\sin \theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right] \mathbf{a}_r + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial(r A_\phi)}{\partial r} \right] \mathbf{a}_\theta + \frac{1}{r} \left[\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \mathbf{a}_\phi$$



Cartesian-Cylindrical Transformations:

Coordinates:

$$\rho = \sqrt{x^2 + y^2} \quad x = \rho \cos \phi$$

$$\tan \phi = \frac{y}{x} \quad y = \rho \sin \phi$$

$$z = z \quad z = z$$

Unit Vectors:

$$\mathbf{a}_\rho = \cos \phi \mathbf{a}_x + \sin \phi \mathbf{a}_y \quad \mathbf{a}_x = \cos \phi \mathbf{a}_\rho - \sin \phi \mathbf{a}_\phi$$

$$\mathbf{a}_\phi = -\sin \phi \mathbf{a}_x + \cos \phi \mathbf{a}_y \quad \mathbf{a}_y = \sin \phi \mathbf{a}_\rho + \cos \phi \mathbf{a}_\phi$$

$$\mathbf{a}_z = \mathbf{a}_z \quad \mathbf{a}_z = \mathbf{a}_z$$

Vector Components:

$$A_\rho = A_x \cos \phi + A_y \sin \phi \quad A_x = A_\rho \cos \phi - A_\phi \sin \phi$$

$$A_\phi = -A_x \sin \phi + A_y \cos \phi \quad A_y = A_\rho \sin \phi + A_\phi \cos \phi$$

$$A_z = A_z \quad A_z = A_z$$

Cartesian-Spherical Transformations:

Coordinates:

$$r = \sqrt{x^2 + y^2 + z^2} \quad x = r \sin \theta \cos \phi$$

$$\tan \theta = \frac{\sqrt{x^2 + y^2}}{z} \quad y = r \sin \theta \sin \phi$$

$$\tan \phi = \frac{y}{x} \quad z = r \cos \theta$$

Unit Vectors:

$$\mathbf{a}_r = \sin \theta \cos \phi \mathbf{a}_x + \sin \theta \sin \phi \mathbf{a}_y + \cos \theta \mathbf{a}_z \quad \mathbf{a}_x = r \sin \theta \cos \phi \mathbf{a}_r + \cos \theta \cos \phi \mathbf{a}_\theta - \sin \phi \mathbf{a}_\phi$$

$$\mathbf{a}_\theta = -\cos \theta \cos \phi \mathbf{a}_x + \cos \theta \sin \phi \mathbf{a}_y - \sin \theta \mathbf{a}_z \quad \mathbf{a}_y = r \sin \theta \sin \phi \mathbf{a}_r + \cos \theta \sin \phi \mathbf{a}_\theta + \cos \phi \mathbf{a}_\phi$$

$$\mathbf{a}_\phi = -\sin \phi \mathbf{a}_x + \cos \phi \mathbf{a}_y \quad \mathbf{a}_z = \cos \theta \mathbf{a}_r - \sin \theta \mathbf{a}_\theta$$

Vector Components:

$$A_r = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta \quad A_x = A_r \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$$

$$A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta \quad A_y = A_r \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$$

$$A_\phi = -A_x \sin \phi + A_y \cos \phi \quad A_z = A_r \cos \theta - A_\theta \sin \theta$$

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{x}A_x + \hat{y}A_y + \hat{z}A_z$	$\hat{r}A_r + \hat{\phi}A_\phi + \hat{z}A_z$	$\hat{R}A_R + \hat{\theta}A_\theta + \hat{\phi}A_\phi$
Magnitude of A $ \mathbf{A} =$	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1,$ for $P = (x_1, y_1, z_1)$	$\hat{r}r_1 + \hat{z}z_1,$ for $P = (r_1, \phi_1, z_1)$	$\hat{R}R_1,$ for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1$ $\hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = \hat{z} \cdot \hat{x} = 0$ $\hat{x} \times \hat{y} = \hat{z}$ $\hat{y} \times \hat{z} = \hat{x}$ $\hat{z} \times \hat{x} = \hat{y}$	$\hat{r} \cdot \hat{r} = \hat{\phi} \cdot \hat{\phi} = \hat{z} \cdot \hat{z} = 1$ $\hat{r} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{z} = \hat{z} \cdot \hat{r} = 0$ $\hat{r} \times \hat{\phi} = \hat{z}$ $\hat{\phi} \times \hat{z} = \hat{r}$ $\hat{z} \times \hat{r} = \hat{\phi}$	$\hat{R} \cdot \hat{R} = \hat{\theta} \cdot \hat{\theta} = \hat{\phi} \cdot \hat{\phi} = 1$ $\hat{R} \cdot \hat{\theta} = \hat{\theta} \cdot \hat{\phi} = \hat{\phi} \cdot \hat{R} = 0$ $\hat{R} \times \hat{\theta} = \hat{\phi}$ $\hat{\theta} \times \hat{\phi} = \hat{R}$ $\hat{\phi} \times \hat{R} = \hat{\theta}$
Dot product $\mathbf{A} \cdot \mathbf{B} =$	$A_x B_x + A_y B_y + A_z B_z$	$A_r B_r + A_\phi B_\phi + A_z B_z$	$A_R B_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A} \times \mathbf{B} =$	$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{x} dx + \hat{y} dy + \hat{z} dz$	$\hat{r} dr + \hat{\phi} d\phi + \hat{z} dz$	$\hat{R} dR + \hat{\theta} R d\theta + \hat{\phi} R \sin \theta d\phi$
Differential surface areas	$ds_x = \hat{x} dy dz$ $ds_y = \hat{y} dx dz$ $ds_z = \hat{z} dx dy$	$ds_r = \hat{r} d\phi dz$ $ds_\phi = \hat{\phi} dr dz$ $ds_z = \hat{z} r dr d\phi$	$ds_R = \hat{R} R^2 \sin \theta d\theta d\phi$ $ds_\theta = \hat{\theta} R \sin \theta dR d\phi$ $ds_\phi = \hat{\phi} R dR d\theta$
Differential volume $dV =$	$dx dy dz$	$r dr d\phi dz$	$R^2 \sin \theta dR d\theta d\phi$

Table 3-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$ $\hat{z} = \hat{z}$	$A_r = A_x \cos \phi + A_y \sin \phi$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r \cos \phi$ $y = r \sin \phi$ $z = z$	$\hat{x} = \hat{r} \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{r} \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{z}$	$A_x = A_r \cos \phi - A_\phi \sin \phi$ $A_y = A_r \sin \phi + A_\phi \cos \phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$ $\hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$	$A_R = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta$ $A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$
Spherical to Cartesian	$x = R \sin \theta \cos \phi$ $y = R \sin \theta \sin \phi$ $z = R \cos \theta$	$\hat{x} = \hat{R} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{R} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_x = A_R \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$ $A_y = A_R \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$
Cylindrical to spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r} \sin \theta + \hat{z} \cos \theta$ $\hat{\theta} = \hat{r} \cos \theta - \hat{z} \sin \theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r \sin \theta + A_z \cos \theta$ $A_\theta = A_r \cos \theta - A_z \sin \theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R \sin \theta$ $\phi = \phi$ $z = R \cos \theta$	$\hat{r} = \hat{R} \sin \theta + \hat{\theta} \cos \theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_r = A_R \sin \theta + A_\theta \cos \theta$ $A_\phi = A_\phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$

$$\int_V \nabla \cdot \mathbf{E} \, dV = \oint_S \mathbf{E} \cdot d\mathbf{s} \quad (\text{divergence theorem}).$$

$$\int_S (\nabla \times \mathbf{B}) \cdot d\mathbf{s} = \oint_C \mathbf{B} \cdot d\mathbf{l} \quad (\text{Stokes's theorem}),$$

$$\nabla^2 \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla \times (\nabla \times \mathbf{E})$$

Integrals

$$\int \frac{dx}{x^2 - a^2} = \begin{cases} \frac{1}{2a} \ln \left(\frac{x-a}{x+a} \right) + c & x^2 > a^2 \\ \frac{1}{2a} \ln \left(\frac{a-x}{a+x} \right) + c & x^2 < a^2 \end{cases}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}) + C$$

$$\int \frac{xdx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2} + C$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x/a^2}{\sqrt{x^2 + a^2}} + C$$

$$\int \frac{xdx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} + C$$

$$\int \frac{x^2 dx}{(x^2 + a^2)^{3/2}} = \ln \left(\frac{\sqrt{x^2 + a^2}}{a} + \frac{x}{a} \right) - \frac{x}{\sqrt{x^2 + a^2}} + C$$

$$\int \frac{dx}{(x^2 + a^2)^2} = \frac{1}{2a^2} \left(\frac{x}{x^2 + a^2} + \frac{1}{a} \tan^{-1} \frac{x}{a} \right) + C$$

$$\int \sqrt{1-x^2} dx = \frac{1}{2} x \sqrt{1-x^2} + \sin^{-1} x$$

$\begin{aligned} \sin(\theta + 2\pi) &= \sin \theta \\ \cos(\theta + 2\pi) &= \cos \theta \\ \tan(\theta + \pi) &= \tan \theta \end{aligned}$

$\begin{aligned} \sin(-\theta) &= -\sin \theta \\ \cos(-\theta) &= \cos \theta \\ \tan(-\theta) &= -\tan \theta \end{aligned}$
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$\begin{aligned} \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta} \\ \sec \theta &= \frac{1}{\cos \theta} & \csc \theta &= \frac{1}{\sin \theta} \end{aligned}$
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$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \end{aligned}$
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$\sin^2 \theta + \cos^2 \theta = 1$

$\sin 2\theta = 2 \sin \theta \cos \theta$
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$\begin{aligned} \cos \theta &= \sin \left(\frac{\pi}{2} - \theta \right) & \sin \theta &= \cos \left(\frac{\pi}{2} - \theta \right) \\ \cot \theta &= \tan \left(\frac{\pi}{2} - \theta \right) & \tan \theta &= \cot \left(\frac{\pi}{2} - \theta \right) \\ \csc \theta &= \sec \left(\frac{\pi}{2} - \theta \right) & \sec \theta &= \csc \left(\frac{\pi}{2} - \theta \right) \end{aligned}$

$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta \end{aligned}$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\begin{aligned}\sin(\pi - \theta) &= \sin \theta \\ \cos(\pi - \theta) &= -\cos \theta \\ \tan(\pi - \theta) &= -\tan \theta\end{aligned}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\sin \alpha \cos \beta = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2}$$

$$\cos \alpha \cos \beta = \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2}$$

$$\sin \alpha \sin \beta = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2}$$

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

$$\sin \theta = \frac{2 \tan \theta/2}{1 + \tan^2 \theta/2}$$

$$\cos \theta = \frac{1 - \tan^2 \theta/2}{1 + \tan^2 \theta/2}$$

$$\tan \theta = \frac{2 \tan \theta/2}{1 + \tan^2 \theta/2}$$